Suggested solutions to the IO (BSc) resit exam on August 15, 2013
VERSION: September 1, 2013

Question 1.

a) **Solve for a subgame perfect Nash equilibrium of the model in which consumers with $r > \widehat{r}$, for some $\widehat{r} \in (0,1)$, consume in period 1. Find the equilibrium value of $\widehat{r}$. Also identify the equilibrium values of p_1 and p_2.**

- We can solve the model by first studying the optimal behavior in period 2 for the firm and the consumers, given some arbitrary cut-off point $\widehat{r} \in (0,1)$. Then, after having found the equilibrium value of p_2 as a function of $\widehat{r}$, we can study the optimal behavior in period 1, thereby identifying the equilibrium values of $\widehat{r}$ and p_1.
- Remember that the monopoly firm is myopic — it cares only about the current period's profit when choosing the current period's price. The consumers, however, care about their future utilities — they use the (common) discount factor δ.

Second period

- Suppose consumers with $v > \widehat{r}$, for some $\widehat{r} \in (0,1)$, consume in period 1.
  - The variable $\widehat{r}$ is of course endogenous and we will later on determine its equilibrium value (in terms of exogenous parameters).
- In period 2, the monopolist then faces the demand schedule

$$q_2 = \widehat{r} - p_2.$$

The derivation of this demand function makes use of the assumption that the r's are uniformly distributed on $[0,1]$ and the fact that the remaining consumers in period 2 buy if and only if their valuation $r \in [0, \widehat{r}]$ exceeds the price p_2. (The students may want to draw a figure to illustrate how the demand function is obtained.)

- The price that maximizes period 2 profits, $\pi_2 = (\widehat{r} - p_2)\, p_2$, is

$$p_2 = \frac{\widehat{r}}{2}. \tag{1}$$

First period

- Given the period 1 price p_1 and the period 2 price $p_2 = \frac{\widehat{r}}{2}$, a consumer will consume in period 1 if and only if

$$r - p_1 \geq \delta\,(r - p_2) = \delta\left(r - \frac{\widehat{r}}{2}\right). \tag{2}$$

Remember that $\widehat{r}$ is defined as the value of r that makes the above inequality hold with equality:

$$\widehat{r} - p_1 = \delta\left(\widehat{r} - \frac{\widehat{r}}{2}\right) \Leftrightarrow \widehat{r} = \frac{2p_1}{2-\delta}. \tag{3}$$

- The firm's profit at the stage when it chooses the period 1 price:

$$\pi_1 = [1 - \widehat{r}]\, p_1 = \left[1 - \frac{2p_1}{2-\delta}\right] p_1.$$

- FOC:

$$\frac{\partial \pi_1}{\partial p_1} = 1 - \frac{4p_1}{2-\delta} = 0$$

  or

$$p_1^* = \frac{2-\delta}{4}. \tag{4}$$

Summing up

- By plugging (4) into (3), we can now get the equilibrium cut-off point

$$\widehat{r}^* = \frac{2p_1^*}{2-\delta} = \frac{1}{2}. \tag{5}$$

- By plugging (5) into (1), we get the equilibrium period 2 price

$$p_2^* = \frac{\widehat{r}^*}{2} = \frac{1}{4}.$$

- At the equilibrium we thus have

$$\boxed{p_1^* = \tfrac{2-\delta}{4} \qquad \text{and} \qquad p_2^* = \tfrac{1}{4},}$$

  and half of the consumers consume in the first period ($\boxed{\widehat{r}^* = \tfrac{1}{2}}$).

b) **Explain in words what the Coase conjecture says. Also explain the intuition.**

- The Coase conjecture concerns a situation where a monopoly firm, in each one of many periods, sells a good that is durable. The firm is allowed to choose a new price in each period. The fact that the good is durable means that those costumers who have bought the good will not need to purchase the good in any future period — these customers disappear from the demand. The Coase conjecture (it was later proven to, under certain conditions, hold as a result) states that:

  – When the length between time periods become smaller (or, equivalently, when the consumers' discount factor approaches one), the monopolist's profit converges to the marginal cost — the firm loses all its market power.

- The reason why this happens is that for any given price in a period, the consumers who find it worthwhile to purchase will be those with the highest valuation. That means that in the next period, those high-valuation consumers are not part of demand and therefore the optimal monopoly price must be lower (since demand is lower). In other words, if the monopoly firm cannot precommit to some sequence of prices but is optimizing in each period given the current demand, the price will gradually drop. However, if the consumers understand this they should have an incentive to wait with purchasing until a later period when the price has fallen. The only thing that may stop the consumers from waiting is that they are impatient and prefer immediate consumption to later, all else being equal. But if the length of time between periods is small or if the consumers are not very impatient (which is the condition in the conjecture), then the consumers don't mind waiting until the price has dropped. If so, the firm may be better off lowering the price straight ahead, so that it doesn't have to wait so long for its (perhaps small) profits.

- To further clarify the explanation we can relate to the result we obtained under a). In that model, whereas the second-period price is constant, *the first-period price is decreasing in the patience parameter δ.* This result is in the spirit of the Coase conjecture, although the monopolist in this simple example doesn't lose all its market power, only some of it.

c) **Define the "Herfindahl index" and the "3-firm concentration ratio". Also, consider a market with seven firms. Their market shares are 5, 5, 10, 10, 20, 20 and 30 percent. Calculate the Herfindahl index and the 3-firm concentration ratio for this market.**

- The Herfindahl index is defined as the sum of the squared market shares, $HI = \sum_{1=1}^{n} s_i$, where s_i is firm i's market share and n is the number of firms in the market.
  - Therefore, the Herfindahl index for this market equals

$$\begin{aligned} HI &= 2 \times \left(\frac{5}{100}\right)^2 + 2 \times \left(\frac{10}{100}\right)^2 + 2 \times \left(\frac{20}{100}\right)^2 + \left(\frac{30}{100}\right)^2 \\ &= \frac{50}{10,000} + \frac{200}{10,000} + \frac{800}{10,000} + \frac{900}{10,000} = \frac{1,950}{10,000} \\ &= 0.195. \end{aligned}$$

- The 3-firm concentration index ratio is defined as the sum of the three largest firms' market shares.
  - Therefore this ratio equals $0.3 + 0.2 + 0.2 = 0.7$.

Question 2a)

- We can solve for the subgame perfect equilibrium by using backward induction. We thus first solve for firm 2's profit-maximizing price, given some fixed price p_1 of firm 1. We then solve firm 1's profit-maximizing problem, assuming that this firm perfectly anticipates firm 2's reaction to any change in p_1.

- Firm 2's profits:

$$\pi_2 = (p_2 - c)\, q_2 = (p_2 - c)\, \frac{1}{3}\, (1 + p_1 - 2p_2)\,.$$

  The first-order condition:

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{3}\left[(1 + p_1 - 2p_2) - 2\,(p_2 - c)\right] = 0.$$

  Solving for p_2 yields

$$p_2^B\,(p_1) = \max\left\{\frac{1 + 2c + p_1}{4}, 0\right\} = \frac{1 + 2c + p_1}{4}.$$

- Firm 1's profits:

$$\pi_1 = (p_1 - c)\, q_1 = (p_1 - c)\, \frac{1}{3}\, (1 - 2p_1 + p_2)\,.$$

  Plugging in $p_2 = p_2^B\,(p_1)$ into that expression yields

$$\begin{aligned}
\pi_1 &= (p_1 - c)\,\frac{1}{3}\left[1 - 2p_1 + p_2^*\,(p_1)\right] \\
&= (p_1 - c)\,\frac{1}{3}\left(1 - 2p_1 + \frac{1 + 2c + p_1}{4}\right) \\
&= \frac{1}{3}\,(p_1 - c)\left(\frac{5 + 2c - 7p_1}{4}\right) \\
&= \frac{1}{12}\,(p_1 - c)\,(5 + 2c - 7p_1)\,.
\end{aligned}$$

  The first-order condition:

$$\frac{\partial \pi_1}{\partial p_1} = \frac{(5 + 2c - 7p_1) - 7\,(p_1 - c)}{12} = 0$$

  or

$$p_1^B = \frac{5 + 9c}{14}.$$

- This yields

$$\begin{aligned}
p_2^B &= p_2^B\left(p_1^B\right) = \frac{1 + 2c + p_1^B}{4} = \frac{1 + 2c + \frac{5+9c}{14}}{4} = \frac{19 + 37c}{4 * 14} \\
&= \frac{19 + 37c}{56}.
\end{aligned}$$

- The subgame perfect equilibrium is thus

$$p_1^B = \frac{5+9c}{14} \qquad \text{and} \qquad p_2^B(p_1) = \frac{1+2c+p_1}{4}.$$

Question 2b)

- As in the previous subquestion we solve for the equilibrium by using backward induction.
- Firm 2's profits are:

$$\pi_2 = (p_2 - c)\, q_2 = (1 - c - q_1 - 2q_2)\, q_2.$$

  The first-order condition is:

$$\frac{\partial \pi_2}{\partial q_2} = 1 - c - q_1 - 4q_2 = 0.$$

  Solving for q_2 yields

$$q_2^C(q_1) = \max\left\{\frac{1-c-q_1}{4}, 0\right\}.$$

- Firm 1's profits:

$$\pi_1 = (p_1 - c)\, q_1 = (1 - c - 2q_1 - q_2)\, q_1.$$

  Plugging in $q_2 = q_2^*(q_1)$ into that expression yields

$$\begin{aligned}
\pi_1 &= [1 - c - 2q_1 - q_2^*(q_1)]\, q_1 \\
&= \begin{cases} \left(1 - c - 2q_1 - \frac{1-c-q_1}{4}\right) q_1 & \text{if } q_1 \leq 1 - c \\ (1 - c - 2q_1)\, q_1 & \text{if } q_1 \geq 1 - c \end{cases} \\
&= \begin{cases} \left(\frac{3(1-c)-7q_1}{4}\right) q_1 & \text{if } q_1 \leq 1 - c \\ (1 - c - 2q_1)\, q_1 & \text{if } q_1 \geq 1 - c. \end{cases}
\end{aligned}$$

- The optimum cannot be such that $q_1 \geq 1 - c$ (the second line) for then profits would be negative. Hence, the first-order condition is:

$$\frac{\partial \pi_1}{\partial q_1} = \frac{3(1-c) - 14q_1}{4} = 0$$

  or

$$q_1^C = \frac{3(1-c)}{14}.$$

- This yields

$$\begin{aligned}
q_2^C &= q_2^C\left(q_1^C\right) = \max\left\{\frac{1-c-q_1^C}{4}, 0\right\} = \frac{1 - c - \frac{3(1-c)}{14}}{4} = \frac{11(1-c)}{4 * 14} \\
&= \frac{11(1-c)}{56}.
\end{aligned}$$

- The subgame perfect equilibrium is thus

$$q_1^C = \frac{3(1-c)}{14} \qquad \text{and} \qquad q_2^C(q_1) = \max\left\{\frac{1-c-q_1}{4}, 0\right\}.$$

Question 2c)

To the external examiner: We have not discussed this result or the intuition behind it in the course. But we have discussed the importance of strategic substitutes/complements for several other results in the IO literature. Below is one attempt to understand the intuition. The important thing is that the students show some ability in understanding the logic of the model.

In the Bertrand game the strategies of the players are strategic complements (i.e., the reaction functions are upward sloping), and in the Cournot game they are strategic substitutes (i.e., the reaction functions are upward sloping). Esther Gal-Or (International Economic Review, 1985) shows, in a two-player setting that is a bit more general than the models in the question, that there is a second-mover in the former case and a first-mover advantage in the latter case. She explains the intuition as follows:

> Downwards sloping reaction functions refer to markets in which the leader can make a preemptive move; upwards sloping reaction functions refer to followers copying or undercutting the leader. An example of the former is when an incumbent firm invests in excess capacity (Spence [1979], Dixit [1980]). Examples of the latter are (i) when an entrant undercuts the price of the incumbent as in the contestable market literature (Baumol [1982]) or (ii) when the follower in the development stage invests more than the leader and is consequently more likely to collect a patent in a research and development game (Reinganum [1983]).

To elaborate a bit on Gal-Or's explanation, we can think of a Bertrand model with a homogeneous good. It is intuitively quite clear that in such a model it should be an advantage to act last — for the second-mover can grab the whole market by slightly undercutting the first mover, after which the first mover is not allowed to make any new move. In the model with differentiated goods, a similar logic is at work, but in a less extreme way.